Angles and the Pythagorean Theorem
CHAPTER 9 Angles and the Pythagorean Theorem

This chapter focuses on several important geometry concepts. You will begin to investigate several concepts in this chapter, but you will learn a lot more about them in future courses.

Section 9.1 focuses on angle relationships. The angle relationships you will explore are the ones found with parallel lines and the angles inside and outside of triangles. You will also learn how you can use what you know about angles to decide if a pair of triangles is similar, without even knowing the side lengths!

In Section 9.2, you will focus on the relationships between side lengths in individual triangles. You will learn how to decide if three different lengths will be able to form a triangle and what kind of triangle will be formed. You will use the unique relationship between the side lengths of right triangles to solve problems.

You will also learn some more about numbers. In particular, you will learn the mathematical operation called “square root” and explore how it relates to squaring a number. You will learn how to convert both terminating and repeating decimals to fractions. Finally, you will look at some special numbers called “irrationals.”

In this chapter, you will learn how to:

- Find the measurements of missing angles made by a line that intersects parallel lines.
- Find unknown angles inside and outside of triangles.
- Determine if two triangles are similar by looking at their angles.
- Find missing side lengths of right triangles using the Pythagorean Theorem.
- Find the square root of a number and identify irrational numbers.
- Convert terminating and repeating decimals to fractions.

Chapter Outline

Section 9.1 You will look at angles formed when a third line intersects a set of parallel lines, identifying the relationships between certain pairs of angles. You will also learn about the special relationships between the angles inside and outside a triangle and how to tell if two triangles are similar without knowing anything about their side lengths.

Section 9.2 You will learn how to determine if any three lengths will form a triangle, and, if they do, whether that triangle will be acute, obtuse, or right. You will find missing sides of right triangles using the Pythagorean Theorem. You will also learn about the square root operation and irrational numbers.
# Chapter 9 Teacher Guide

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|         | 9.1.2  | 1    | Finding Unknown Angles in Triangles    | • Lesson 9.1.2A-D Res. Pgs.  
• Scissors  
• Computer with projector, or paper triangles | 9-21 to 9-26 |
|         | 9.1.3  | 1    | Exterior Angles in Triangles | • Lesson 9.1.3 Res. Pg. | 9-33 to 9-38 |
|         | 9.1.4  | 1    | AA Triangle Similarity | • Straightedges or rulers | 9-545 to 9-50 |
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**Chapter Closure**  
Various Options  

**Total:** 14 days plus optional time for Closure and Assessment
9.1.1 How are the angles related?

Parallel Line Angle Pair Relationships

In a previous course you probably learned the vocabulary and considered the relationships created by two intersecting lines. Now you will look at the vocabulary and relationships created by a line that intersects two parallel lines.

9-1. The box below has some reminders about notation. Read the information, and then use it to complete the following problems.

Arrowheads at the end of lines indicate that they extend indefinitely. Marks on pairs of lines or segments like > and >> indicate that the lines (or segments) are parallel.

The small box at the point of intersection of two lines or segments indicates that the lines (or segments) are perpendicular (that is, form right angles).

In Figure 1 below, line $s$ is parallel to line $t$, and line $r$ intersects (cuts) line $s$ and line $t$. In Figure 2, lines $x$ and $y$ are not parallel, and line $w$ intersects lines $x$ and $y$. Lines $r$ and $w$ are called transversals because they cut across (intersect) 2 lines. Transversals can also intersect several lines, each at different points.

![Figure 1 and Figure 2 diagrams]

a. Take a sheet of plain paper (tracing paper is best, but binder paper will work) and place it over $\angle a$. Using a ruler as a guide, make a precise copy of the angle. Slide the copy of $\angle a$ to $\angle c$ and compare the sizes of the two angles. What do you observe?

b. Trace a copy of $\angle b$ and place it over $\angle d$. What do you observe?

c. Trace a copy of $\angle e$ and place it over $\angle g$. What do you observe?

d. Trace a copy of $\angle f$ and place it over $\angle h$. What do you observe?

e. Both figures show two lines that are intersected by a transversal. Summarize your findings by describing the relationship between pairs of angles and parallel lines cut by a transversal. That is, when the angles are congruent, what must be true? And vice versa: If the lines are parallel, then what must be true?
9-2. Your teacher will provide you with a Lesson 9.1.1 Resource Page. On it, you will find the figures below. Keep it in an easily accessible place, as you will use it in several problems.

Each figure below shows a pair of parallel lines, \( p \) and \( q \), which are intersected (cut) by a third line, \( m \). Line \( m \), often called a transversal, forms several angles at each point of intersection with \( p \) and \( q \). If you need help with some of the vocabulary from a previous course, see the Math Notes box in this lesson.

![Figure 1](image1)

![Figure 2](image2)

![Figure 3](image3)

**Figure 1**  
**Figure 2**  
**Figure 3**

a. Use what you know about straight angles to calculate the measures of these angles: \( b \), \( d \), \( f \), \( k \), \( r \), and \( s \). When everyone in your team has completed the calculations and agrees with the results, check with your team to be sure that everyone agrees that the results are correct.

b. Keep in mind that lines \( p \) and \( q \) must be parallel as you complete the directions below.

- In Figure 1, compare the measures of angles \( a \) and \( d \), and then compare the measures of angles \( b \) and \( e \).
- In Figure 2, compare the measures of angles \( f \) and \( j \), and then compare the measures of angles \( g \) and \( k \).
- In Figure 3, make similar comparisons for angles \( n \) and \( s \), and then for angles \( r \) and \( t \).

Angles on the same side of two lines and on the same side of a third line (the transversal) that intersects the two lines are called corresponding angles. In the figure at right, angles 1 and 2 are corresponding angles, as are angles 3 and 4. Other examples of corresponding angles are on your resource page: angles \( a \) and \( d \) in Figure 1, angles \( g \) and \( k \) in Figure 2, and angles \( r \) and \( t \) in Figure 3.

c. A conjecture is an inference or judgment based on incomplete evidence. Use the definition above and your observations in part (b) to complete the following conjecture.

**Conjecture:** If two parallel lines are cut by a transversal, then pairs of corresponding angles are ___.

a. Use what you know about straight angles and/or vertical angles and your results from the previous problem to find the measures of angles $e$ (Figure 1), $h$ (Figure 2), and $w$ (Figure 3).

b. Compare the measures of the following three pairs of angles.
   - Figure 1: $m\angle e$ and $m\angle d$
   - Figure 2: $m\angle h$ and $m\angle j$
   - Figure 3: $m\angle w$ and $m\angle s$

   How is each pair of angles related?

c. Read the following definition, and then use it along with your observation in part (a) to complete the conjecture that follows.

```
Angles between a pair of lines and on opposite sides of a transversal are called alternate interior angles.
In the figure at right, angles 1 and 2 and angles $x$ and $y$ are examples of pairs of alternate interior angles.
Other examples of alternate interior angles are on your resource page: angles $e$ and $d$ in Figure 1, angles $h$ and $j$ in Figure 2, and angles $w$ and $s$ in Figure 3.
```

**Conjecture:** If parallel lines are cut by a transversal, then alternate interior angles are ____.

a. Examine the pairs of angles $b$ and $d$, $g$ and $j$, and $r$ and $s$ on the resource page. If you add the measures of each pair, what do you observe?

b. Write a conjecture about two angles on the same side of a transversal and are between two parallel lines. Note: These are called **same side interior angles**.

**Conjecture:** The sum of the measures of two interior angles on the same side of a transversal is _____.

Use your conjecture and the figure below right to answer parts (c) and (d).

- If $m\angle 2 = 67^\circ$, what is $m\angle 5$?
- If $m\angle 4 = 4x + 23^\circ$ and $m\angle 6 = 3x + 17^\circ$, find $m\angle 4$. Explain your steps.
9-5. Classify each of the following pairs of angles as corresponding, alternate interior, same side interior, straight, or "none of these."

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

i. 

j. What condition is necessary to be able to say that the pairs of corresponding angles or alternate interior angles above are equal?
9-6. Use your conjectures about parallel lines and the angles formed by a transversal to find the measures of the labeled angles. These figures are also on the Lesson 9.1.1 Resource Page. Show the step-by-step procedure you use and name each angle conjecture you use (e.g., corresponding, alternate interior, vertical, or straight) to justify your calculation.

a. \[ \begin{align*}
\angle a & = \angle p \\
\angle b & = \angle q \\
\angle c & = 123^\circ
\end{align*} \]

b. \[ \begin{align*}
\angle d & = \angle e \\
\angle f & = 82^\circ
\end{align*} \]

c. \[ \begin{align*}
\angle m & = 75^\circ
\end{align*} \]
9-7. LEARNING LOG

What are the angle relationships that you have encountered so far? Answer this question in your Learning Log. Use your geometry vocabulary and include diagrams to show the relationships. Title this entry “Angle Relationships” and label it with today’s date.
METHODS AND MEANINGS

Angle Vocabulary

It is common to identify angles using three letters. For example, \( \angle ABC \) means the angle you would find by going from point \( A \) to point \( B \) to point \( C \) in the diagram at right. Point \( B \) is the vertex of the angle (where the endpoints of the two sides meet), and \( BA \) and \( BC \) are the rays that define it. A ray is a part of a line that has an endpoint (starting point) and extends infinitely in one direction.

If two angles have measures that add up to 90°, they are called complementary angles. For example, in the diagram above, \( \angle ABC \) and \( \angle CBD \) are complementary because they form a right angle.

If two angles have measures that add up to 180°, they are called supplementary angles. For example, in the diagram at right, \( \angle EFG \) and \( \angle GFH \) are supplementary because together they form a straight angle (that is, together they form a line).

Two angles do not have to share a vertex to be complementary or supplementary. The first pair of angles at right are supplementary; the second pair of angles are complementary.

Adjacent angles are angles that have a common vertex, share a common side, and have no interior points in common. So angles \( \angle c \) and \( \angle d \) in the diagram at right are adjacent angles, as are \( \angle c \) and \( \angle f \), \( \angle f \) and \( \angle g \), and \( \angle g \) and \( \angle d \).

Vertical angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines, such as angles \( \angle c \) and \( \angle g \) in the diagram above right. By itself, \( \angle c \) is not a vertical angle, nor is \( \angle g \), although \( \angle c \) and \( \angle g \) together are a pair of vertical angles. Vertical angles always have equal measure.
9-8. Use the conjectures and definitions in this lesson to solve parts (a) and (b). Each part is a separate problem.
   a. If $m\angle 1 = 63^\circ$, find $m\angle 2$ and $m\angle 3$ by calculation.
   b. If $m\angle 1 = 74^\circ$ and $m\angle 4 = 3x - 18^\circ$, write an equation and find $x$.
   c. If $m\angle 2 = 3x - 9^\circ$ and $m\angle 1 = x + 25^\circ$, write an equation to find $x$. Then find $m\angle 2$.

9-9. If $m\angle 5 = 53^\circ$ and $m\angle 7 = 125^\circ$, find the measures of each numbered angle. Then explain how you found each angle, citing definitions and conjectures that support your steps.

9-10. Graph the following data as a scatterplot.

<table>
<thead>
<tr>
<th>Height (inches)</th>
<th>Test Scores (Percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>83</td>
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<tr>
<td>56</td>
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<td>27</td>
<td>77</td>
</tr>
<tr>
<td>64</td>
<td>88</td>
</tr>
</tbody>
</table>

9-11. Jonas called his father to say that he was almost home. He had traveled 61.5 miles, which was $\frac{3}{4}$ of the way home. Write and solve an equation to calculate the total distance he will travel to get home.

9-12. Ryan and Janelle are each driving from a different location to meet. When they each stopped for lunch at 12 noon, they called each other on their cell phones. Ryan had traveled 245 miles in 3$\frac{1}{2}$ hours. Janelle had driven 290 miles in 4 hours.

If Ryan and Janelle were originally 910 miles apart when they had started driving that morning, at what time will they meet?

9-13. Determine whether the graphs below are functions or not functions. Explain your reasoning.

a. 

b. 

c. 

d. 

e. 

f. 

Lesson 9.1.1 Resource Page

9-2.

\[107^\circ = a\]
\[73^\circ = c\]

![Figure 1](image1)

\[g = 152^\circ\]
\[j = 28^\circ\]

![Figure 2](image2)

9-5.

\[41^\circ = n\]
\[r = s\]
\[t = 139^\circ\]

![Figure 3](image3)

9-6.

\[d\]
\[e\]
\[j = 82^\circ\]

![Figure 4](image4)

9-5.

\[a\]
\[b\]
\[c = 123^\circ\]

![Figure 5](image5)

9-6.

\[g\]
\[h\]
\[q\]

![Figure 6](image6)
9.1.2 How can I find a missing angle?

Finding Unknown Angles in Triangles

In today’s lesson, you will be challenged again to use what you do know to determine information that you did not previously know, in order to solve problems with variables. You will do an investigation to learn a new geometric relationship for triangles.

9.14. Quigley was excited about what he had learned about angles. He went home, grabbed his older brother’s math book, and tried to find some problems that he could do with angles. He came across the following problem that he wanted to solve.

*Solve for x in this figure:*

```
41°  x  62°
```

a. Using what you have learned about angles, can you find the measure of the angle? Why or why not?

b. Estimate the measure of the angle.
9.15. TANGLED TRIANGLES

Your teacher will give your team a copy of the Lesson 9.1.2 Resource Page. Cut out the three copies of the triangle.

Your Task: Determine the measure of the missing angle without using a protractor. As you work with your team, the following questions might help guide your discussion.

What do we know about angles?

Can we combine the unknown angle with any other angles to create a new angle that we do know?
9.16. Be prepared to contribute what your team has discovered to a whole-class discussion. Your teacher will use a technology tool to show what each team has discovered for their triangle. Keep track of what each team has found to see if you can find a relationship that would allow you to find a missing angle in any triangle.
9-17. Now use what you have discovered about the angles in a triangle to find the answer to the problem that Quigley was trying to solve in problem 9-14. How close was your estimate?
9-18. Use what you have learned about triangles and angles to write an equation that represents each situation. Then find each of the missing angle(s) in the triangles below.

a. \[
\begin{array}{c}
30^\circ \\
115^\circ \\
\end{array}
\]

b. \[
\begin{array}{c}
x \\
x \\
x \\
x \\
x \\
x \\
x \\
x \\
x \\
x \\
\end{array}
\]
9-19. **Additional Challenge:** Use what you know about triangles and angle relationships to find the missing angles in the triangles below.

a. 

\[ \begin{array}{c}
2x \\
30^\circ
\end{array} \]

b. 

\[ \begin{array}{c}
x + 10^\circ \\
90^\circ \\
x
\end{array} \]

c. 

\[ \begin{array}{c}
36^\circ \\
x \\
92^\circ \\
116^\circ \\
15^\circ \\
y
\end{array} \]

d. 

\[ \begin{array}{c}
x \\
105^\circ \\
54^\circ \\
y
\end{array} \]
9-20. LEARNING LOG

Today you explored a fundamental concept about a triangle and the sum of its angles. In your Learning Log, state this relationship in your own words and include at least one example that shows how to use this idea. Title this entry "Angles in a Triangle" and label it with today's date.
**Methods and Meanings**

Parallel Lines and Angle Pairs

**Math Notes**

**Corresponding angles** lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\angle m$ and $\angle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.

$\angle f$ and $\angle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.

$\angle g$ and $\angle m$ are **same side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same side interior angles are supplementary when the lines intersected by the transversal are parallel.
9-21. Find the measure of the missing angle in each triangle below and then classify the triangle as acute, right, or obtuse.

a.  
\[
\begin{array}{c}
35^\circ \\
\angle \end{array}
\]

b.  
\[
\begin{array}{c}
65^\circ \\
55^\circ \\
x \\
\angle \\
\end{array}
\]

9-22. Find the measures of the angles requested and explain how you found them. Each part is a separate problem.

a. If  \( m\angle 4 = 61^\circ \), find  \( m\angle 6 \).

b. If  \( m\angle 1 = 48^\circ \), find  \( m\angle 8 \).

c. If  \( m\angle 2 = 137^\circ \), find  \( m\angle 8 \).

9-23. Graph the rule  \( y = 2x - 6 \). Create a table if it will help.

9-24. On graph paper, graph the data in the table at right. Is there an association?

<table>
<thead>
<tr>
<th>Age of Car (years)</th>
<th>Avg. Miles per Gallon</th>
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</thead>
<tbody>
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<td>7</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
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<tr>
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<td>29</td>
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<tr>
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<td>3</td>
<td>35</td>
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<tr>
<td>8</td>
<td>18</td>
</tr>
</tbody>
</table>

9-25. Tina’s rectangular living room floor measures 15 feet by 18 feet.

a. How many square feet of carpet will Tina need to cover the entire floor?

b. The carpet Tina likes is sold by the square yard. How many square yards will she need?

9-26. Solve for \( y \) in terms of \( x \). That is, rewrite each equation so that it starts with "\( y = \)".

a.  \( 6x + 5y = 20 \)

b.  \( 4x - 8y = 16 \)
Lesson 9.1.2D Resource Page

A

\[ s \]

\[ 60^\circ \]

\[ 80^\circ \]

B

\[ s \]

\[ 60^\circ \]

\[ 80^\circ \]

C

\[ s \]

\[ 60^\circ \]

\[ 80^\circ \]
9.1.3 What if the angle is on the outside?

Exterior Angles in Triangles

So far in this section, you have investigated angle relationships in situations with parallel lines and within triangles. In this lesson, you will continue to look at angle relationships. This time, you will investigate the angle relationships on the inside and outside of a triangle.

9-27. Use the diagram at right to name each of the indicated angles below using three letters. Reread the Math Notes box in Lesson 9.1.1 if you need to remember how to do this.

a. \( \angle a \)  
b. \( \angle b \)  
c. \( \angle c \)
9.28. Read the information below. Then follow the directions that follow.

**Exterior angles** are formed by extending a side of the triangle. The two angles across the triangle from the exterior angle are called **remote interior angles.** In each figure located below part (a), \( \angle A \) and \( \angle B \) are remote interior angles with respect to exterior angle \( \angle BCD \).

These are exterior angles: ![Diagram of exterior angles]

These are NOT exterior angles: ![Diagram of non-exterior angles]

a. On the Lesson 9.1.3 Resource Page provided by your teacher, calculate the missing angle measures in each figure and record them in the table. When you have completed the resource page, look for a pattern in the relationship between the measure of \( \angle BCD \) (the exterior angle) and the sum of the measures of \( \angle A \) and \( \angle B \) (the remote interior angles). (Note that \( m\angle BCD \) means the **measure** of \( \angle BCD \). Similarly, \( m\angle A \) means the measure of \( \angle A \).)

![Figures 1 to 4]

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>( m\angle A )</th>
<th>( m\angle B )</th>
<th>( m\angle ACB )</th>
<th>( m\angle BCD )</th>
<th>( m\angle A + m\angle B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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</table>

b. Compare your results for \( m\angle BCD \) (the exterior angle) and the sum of \( m\angle A \) and \( m\angle B \) (the remote interior angles) for each figure. Discuss your observations with your team.

c. Write a conjecture about the relationship of an exterior angle to the two remote interior angles.
9-29. Calculate the measures of the angles requested. Each part is a separate problem.

a. If $m\angle 1 = 53^\circ$ and $m\angle 2 = 71^\circ$, find $m\angle 4$.

b. If $m\angle 2 = 78^\circ$ and $m\angle 4 = 127^\circ$, find $m\angle 1$. 

\[ \text{Diagram of a triangle with angles 1, 2, 3, and 4.} \]
9-30. Use your conjecture from part (c) of problem 9-28 to solve for $x$ in each figure below.

a. \[ \begin{align*}
2x & \quad 3x \\
& \quad 135^\circ
\end{align*} \]

b. \[ \begin{align*}
3x - 1 & \quad 21^\circ \\
& \quad 167^\circ
\end{align*} \]

c. \[ \begin{align*}
33^\circ & \quad 41^\circ \\
& \quad 2x + 30
\end{align*} \]
9-31. **Additional Challenge:** Solve for $x$ in the figure at right.
9-32. LEARNING LOG

Look back at the entry that you created in Lesson 9.1.1 ("Angle Relationships"). Add to that entry the angle relationships that you learned in this lesson. Use appropriate geometry vocabulary and include diagrams.
MATH NOTES

METHODS AND MEANINGS

Angle Sum Theorem for Triangles

The measures of the angles in a triangle add up to 180°. For example, in \( \triangle ABC \) at right, 
\[ m\angle A + m\angle B + m\angle C = 180°. \]

You can verify this statement by carefully drawing a triangle with a ruler, tearing off two of the angles (\( \angle A \) and \( \angle B \)), and placing them side by side with the third angle (\( \angle C \)) on a straight line. The sum of the three angles is the same as the straight angle (line), that is, 180°.
9-33. Based on the given information, determine which pairs of lines, if any, are parallel. If none are necessarily parallel, write “none.”

a. \( m \angle 2 = m \angle 7 \)

b. \( m \angle 3 = m \angle 11 \)

c. \( m \angle 1 = m \angle 12 \)

d. \( m \angle 13 = m \angle 12 \)

e. \( \angle 6 \) and \( \angle 7 \) are supplementary.

9-34. In each angle problem below, solve for the variable(s). Write the names of the definition(s) and relationship(s) that justify the steps in your solution.

a.  
\[ 132^\circ = 2x + 3^\circ \]

b.  
\[ 131^\circ = d \]

c.  
\[ 19^\circ + 24^\circ \]

9-35. Simplify each expression.

a. \( \frac{5}{4} + \frac{7}{16} \)

b. \( -\frac{10}{13} \cdot \frac{5}{11} \)

c. \( \frac{9}{11} + \left(-\frac{20}{21}\right) \)

d. \( -\frac{8}{3} + \left(-\frac{5}{18}\right) \)

9-36. Simplify and solve each equation below for \( x \). Show your work and check your answer.

a. \( 24 = 3x + 3 \)

b. \( 2(x - 6) = x - 14 \)

c. \( 3(2x - 3) = 4x - 5 \)

d. \( \frac{3}{4}x = 2x - 5 \)

9-37. Joaquin has agreed to lend his younger brother $45 so that he can buy a new tank for his pet lizard.

a. Joaquin is charging his brother 2% simple interest per month. If his brother pays him back in 6 months, how much will Joaquin get back?

b. If Joaquin’s brother instead borrowed the money from a bank at 2% compound interest per month, how much would he have to pay the bank at the end of 6 months?

9-38. Martha is saving money to buy a new laptop computer that costs $1800. She received $200 for her birthday and has a job where she makes $150 each week.

a. Make a table and a graph for this situation.

b. Explain how you can use the table or graph to predict how many weeks it will take Martha to earn enough money to pay for the new computer.

c. Explain how you can tell from both the table and the graph whether this is an example of linear or non-linear growth.
Lesson 9.1.3 Resource Page

Calculate the missing angle measures in each figure and record them in the table.

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>$m \angle A$</th>
<th>$m \angle B$</th>
<th>$m \angle ACB$</th>
<th>$m \angle BCD$</th>
<th>$m \angle A + m \angle B$</th>
</tr>
</thead>
<tbody>
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</table>
9.1.4 Can angles show similarity?

AA Triangle Similarity

In Chapter 6 you learned that you can create similar figures by using dilations. Today you will investigate what happens to the angles in a figure when you enlarge or reduce the figure to create a similar figure.

9-39. ANGLES IN SIMILAR FIGURES

a. Using a sheet of graph paper and a straightedge, graph the quadrilateral $M(0, 3), N(4, 0), P(2, -2), Q(-2, 1)$.

b. Enlarge the quadrilateral by a scale factor of 2.

c. What do you notice about side $MN$ and side $M'N'$? Explain.

d. What can you say about $\angle M$ and $\angle M'$? Explain your reasoning. Hint: Extend sides $MN$ and $QM$.

e. Remember that a conjecture is an inference or judgment based on incomplete evidence. Based on your work in this problem so far, make a conjecture about the angles in similar figures.

f. Test your conjecture in part (e) using a figure of your own design and a different scale factor. Each team member should create a different figure. Compare your work with your teammates’ work. Does your conjecture seem to work always, sometimes, or never?
9-40. Imagine that two pairs of corresponding angles in two triangles are of equal measure. What could you then conclude about the third set of angles? Justify your answer and draw a diagram.
9.41. Use your conjecture from part (e) of problem 9-39 along with your work from problem 9-40 to explain how you can use the angles in a pair of triangles to determine if they are similar. Be sure to include how many angles you need and what needs to be true about them.
9-42. The relationship in the previous problem is called **Angle-Angle Similarity** and is written $AA\sim$. The symbol $\sim$ means "similarity" or "is similar to." In the figure at right, is $\triangle ABC \sim \triangle EDC$ (that is, is $\triangle ABC$ similar to $\triangle EDC$)? Explain your reasoning.
Eleanor and John were working on a geometry problem together. They knew that in the figure below, line $m$ is parallel to side $BC$. They wanted to find the side lengths of each triangle. First they decided that they needed to show that $\triangle AED \sim \triangle ABC$.

Eleanor said, "This is easy. We have parallel lines, so the triangles are similar by AA\~."

"Hold on a minute!" John replied, "Which angles are equal?"

a. Using the diagram at right, name the pairs of equal angles Eleanor sees. Why are they equal?

b. Are the triangles ($\triangle AED$ and $\triangle ABC$) similar? Explain.

c. Now that John sees how the triangles are similar, he suggests redrawing them separately as shown at right. "Look," he says, "Now we just write a proportion."

He suggests the following equation:

$$\frac{x}{3+5} = \frac{x}{x+8}$$

Explain how John came up with this equation.

d. Solve the proportion equation in part (c) for $x$ and check your answer.
What is Angle-Angle Similarity? What does it tell you about a pair of triangles? In your Learning Log, explain the relationship and how you can use it. Be sure to include a diagram. Title this entry “Angle-Angle Similarity” and include today’s date.
An exterior angle of a triangle is an angle outside of the triangle created by extending one of the sides of the triangle. In the diagram at right, $\angle 4$ is an exterior angle. The Exterior Angle Theorem for Triangles states that the measure of the exterior angle of a triangle is equal to the sum of the remote interior angles. In the diagram, $\angle 1$ and $\angle 2$ are the remote interior angles to $\angle 4$. Note that some texts call these angles “opposite interior angles.” In symbols:

$$m\angle 4 = m\angle 1 + m\angle 2$$
9-45. \( \triangle ABC \) is similar to \( \triangle DEF \).
   a. Find the scale factor from \( \triangle ABC \) to \( \triangle DEF \).
   b. Find \( x \).
   c. Find \( y \).

9-46. Sketch an example of each type of graph described below.
   a. linear and decreasing
   b. non-linear and increasing

9-47. Louis recorded how many times he could jump rope without stopping. Here is his data:
   
   50 15 64 29 55 100 97 48 81 61

   Find the median, first quartile, and third quartile of his data.

9-48. Solve the system of equations at right by each of the ways described in parts (a) and (b) below. Then compare your answers in part (c).
   a. Graph the system on graph paper. Then write its solution (the point of intersection) in \((x, y)\) form.
   \[ y = -x + 1 \]
   \[ y = 2x + 7 \]
   b. Now solve the system using the Equal Values Method.
   c. Did your solution in part (b) match your result from part (a)? If not, check your work carefully and look for any mistakes in your algebraic process or on your graph.

9-49. Use what you know about the angles in a triangle to find \( x \) in each diagram below. Show all work. Then classify each triangle as acute, right, or obtuse.
   a. \[ \triangle \]
   b. \[ \triangle \]

9-50. This problem is a checkpoint for scatterplots and association. It will be referred to as Checkpoint 9.

   Jason is interested in buying a used Puda hybrid car because he heard of its incredible gas mileage. Jason collected data from listings on the Internet.
   a. Jason would like to know the typical cost of advertised Pudas. What kind of a graphical display should he use?
   b. What type of graph should be used to display the relationship between age and cost?
   c. Make a scatterplot of the data.
   d. Fully describe the association.
   e. Draw a line of best fit for the data. Find the equation of the line of best fit.
   f. Use the equation to predict the expected cost of a 6-year-old car.
   g. Interpret the slope and \( y \)-intercept in context.

Check your answers by referring to the Checkpoint 9 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 9 materials and try the practice problems. Also consider getting help outside of class time. From this point on, you will be expected to do problems like this one quickly and easily.
9.2.1 What kind of triangle can I make?

Side Lengths and Triangles

Triangles are made of three sides. But can any three lengths make a triangle? Is it possible to predict what kind of triangle – obtuse, acute, or right – three side lengths will make? Today, you will investigate these questions with your team.

9-51. IS IT A TRIANGLE?

When do three lengths form a triangle? Are there special patterns in lengths that always make obtuse triangles? Or are there combinations of lengths that make right or acute triangles? To investigate these questions, you and your study team will build triangles and look for patterns that will allow you to predict what kind of triangle three lengths will make.

Discussion Points

How can we organize our data?

What other combinations can we make?

What do we expect will happen?

Your Task: Your teacher will provide you with three resource pages for this activity. Carefully cut out each square on the Lesson 9.2.1A Resource Page, shown at right. Using different combinations of three squares, decide if a triangle can be made by connecting the corners of the squares. If you can make a triangle, what kind of triangle is it? (The angle on the Lesson 9.2.1B Resource Page can help you determine if an angle is a right (90°) angle.) Record the side lengths and areas for each combination of squares you try on the Lesson 9.2.1C (or D) Resource Page. Complete the other columns of the chart.
9-52. **Look at your data for the combinations that did not form triangles.**

- What do you notice about how the three side lengths compare to each other?
- How are the sets of three side lengths that did not form triangles different from the sets of side lengths that *did* form triangles? Be as specific as you can.

When your team has reached a conclusion, copy and complete the two statements below in your Learning Log. Title your entry "Triangle Inequality" and include examples and today’s date.

*Three side lengths WILL NOT make a triangle if...*

*Three side lengths WILL make a triangle if...*
Look at the data that you collected for the acute, obtuse, and right triangles. What patterns do you see between the sum of the areas of the two smaller squares and the area of the larger square that formed the triangles in each row? Copy and complete the sentence starters below in your Learning Log to summarize the patterns that you see. Title the entry “Triangle Side-Length Patterns” and include some examples along with the date.

*If three squares have sides that make an acute triangle, then the sum of the areas of the two small squares...*

*If three squares have sides that make an obtuse triangle, then the sum of the areas of the two small squares...*

*If three squares have sides that make a right triangle, then the sum of the areas of the two small squares...*
9-54. Use the patterns you found to predict whether each set of lengths below will form a triangle. If a set will form a triangle, state whether the triangle will be acute, obtuse, or right. Justify your conclusion.

a. 5 cm, 6 cm, and 7 cm
b. 2 cm, 11 cm, 15 cm

c. 10 cm, 15 cm, 20 cm
d. 10 cm, 24 cm, 26 cm

e. 1 cm, 3 cm, 9 cm
f. 2 cm, 10 cm, 11 cm
9-55. **Additional Challenge:** Lewis wants to build an obtuse triangle. He has already decided to use a square with an area of 81 square units and a square with an area of 25 square units.

a. What area of square could he use to form the third side of his triangle? Explain your reasoning.

b. If he makes an acute triangle instead, what size square should he use? Explain your reasoning.
METHODS AND MEANINGS

AA Similarity for Triangles

For two triangles to be similar, corresponding angles must have equal measure.

However, it is sufficient to know that two pairs of corresponding angles have equal measures, because then the third pair of angles must have equal measure.

This is known as the Angle-Angle Triangle Similarity Conjecture, which can be abbreviated as “AA Similarity” or “AA ~.”
9-56.  a. Are the two triangles at right similar? How do you know?

b. Find \( x \). Show your strategy.

c. What is the scale factor (\[
\frac{\text{big}}{\text{small}}\])?

d. Find the area of each triangle.

9-57. Jenna is working with three squares. Their areas are 16 cm\(^2\), 9 cm\(^2\), and 36 cm\(^2\). She thinks they will make an obtuse triangle. Do you agree? Explain your reasoning.

9-58. Copy and complete the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>5</th>
<th>4</th>
<th>-2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-17</td>
<td>-5</td>
<td>4</td>
<td>-11</td>
</tr>
</tbody>
</table>

a. What is the rule?

b. What is the slope?

9-59. Write an equation to represent the situation below, and then answer the question.

Ella is trying to determine the side lengths of a triangle. She knows that the longest side is three times longer than the shortest side. The medium side is ten more than twice the shortest side. If the perimeter is 142 cm, how long is each side?

9-60. Cisco was looking at a table of values for the rule \( y = x^2 \). She said, "This table contains \((0, 0)\), so I think it shows a proportional relationship." Is Cisco correct? Why or why not?

9-61. Solve these equations for \( x \). Check your answers.

a. \( 2(x + 4.5) = 32 \)  
b. \( 6 + 2.5x = 21 \)  
c. \( \frac{x}{5} = \frac{5}{18} \)

9-62. What kind of triangle will the edges of the squares at right form? What will the side lengths be?
9-63. Determine by inspection whether the lines in each system below intersect, coincide, or are parallel. Do not actually solve the systems. Justify your reasons.

   a. \( y = 2x + 3 \)
   \( y = \frac{1}{2} x - 2 \)
   b. \( 2x + 3y = 6 \)
   \( 2x + 3y = 9 \)
   c. \( y = \frac{1}{3} x + 2 \)
   \( y = \frac{1}{3} x - 2 \)
   d. \( x - 2y = 4 \)
   \( -2x + 4y = -8 \)

9-64. Use the graph below to answer the following questions.

   a. What kind of growth does this graph show? How do you know?
   b. What is this graph describing? Write an appropriate title for the graph.
   c. How far from home is the person when the graph starts?
   d. How fast is the person traveling? Explain how you can use the graph to determine the rate of travel.
   e. Write an equation to represent the line on the graph.

9-65. Eric set up this ratio for two similar triangles:
\[
\frac{5}{12} = \frac{5}{9}
\]
He solved the problem and found \( x = 6.67 \). What was his mistake?

9-66. If one atom of carbon weighs \( 1.99 \times 10^{-22} \) g and one atom of hydrogen weighs \( 1.67 \times 10^{-27} \) g, which element weighs more? Explain your choice.

9-67. Andrea wants to have $9500 to travel to France when she is 22. She currently has $5976 in a savings account earning 5% annual compound interest. Andrea is 14 now.

   a. If she does not take out or deposit any money, how much money will Andrea have when she is 18?
   b. Will Andrea have enough money for her trip when she is 22?
Lesson 9.2.1A Resource Page

Is It a Triangle?

How many different triangles can you make using these squares?

\[ 5^2 = 25 \]
\[ 6^2 = 36 \]
\[ 13^2 = 169 \]
\[ 10^2 = 100 \]
\[ 12^2 = 144 \]
\[ 4^2 = 16 \]
\[ 8^2 = 64 \]
\[ 3^2 = 9 \]
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<th>Length</th>
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<th>Small Area + Medium Area</th>
<th>Equal (=)</th>
<th>Greater than (&gt;)</th>
<th>Less than (&lt;)</th>
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<th>Acute Triangle</th>
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### Data Page

#### Acute Triangles

<table>
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<tr>
<th>Side Length</th>
<th>A</th>
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</table>

If three squares have sides that make an acute triangle, then the sum of the areas of the two small squares...

#### Obtuse Triangles

<table>
<thead>
<tr>
<th>Side Length</th>
<th>A</th>
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<th>D</th>
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</table>

If three squares have sides that make an obtuse triangle, then the sum of the areas of the two small squares...

#### Right Triangles

<table>
<thead>
<tr>
<th>Side Length</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
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<td>S</td>
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If three squares have sides that make a right triangle, then the sum of the areas of the two small squares...

#### Not Triangles

<table>
<thead>
<tr>
<th>Side Length</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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Three side lengths WILL NOT make a triangle if...

Three side lengths WILL form a triangle if...
<table>
<thead>
<tr>
<th>Length Small Side</th>
<th>Length Medium Side</th>
<th>Length Largest Side</th>
<th>Area of Small Square</th>
<th>Area of Medium Square</th>
<th>Area of Large Square</th>
<th>Small Area + Medium Area</th>
<th>Equal ( = ) Greater than ( &gt; ) Less than ( &lt; )</th>
<th>Large Area</th>
<th>Acute Triangle</th>
<th>Obtuse Triangle</th>
<th>Right Triangle</th>
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<td>144</td>
<td>169</td>
<td>208</td>
<td>&gt;</td>
<td>169</td>
<td>Acute Triangle</td>
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All other side combinations

Sum of smaller areas < Large area

Obtuse triangle
9.2.2 What is special about a right triangle?

Pythagorean Theorem

In Lesson 9.2.1, you saw that for three lengths to form a triangle, they must be related to each other in a special way. Today, you will investigate a special relationship between the side lengths of right triangles. This relationship will allow you to find the length of a missing side.

9-68. Use your patterns from Lesson 9.2.1 to decide if the squares listed below will form a right triangle.

a. Squares with side lengths 6, 8, and 10 meters
b. Squares with areas 64 in², 100 in², 144 in²
c. Two squares with side length 5 feet and a square with area 50 square feet
d. Explain how you know whether three squares will join at their corners to form a right triangle.
THE PYTHAGOREAN RELATIONSHIP

Based on your work so far, if you know the area of three squares, you can tell if they will connect at their corners to form a right triangle. But what if you know that a triangle has a right angle? Will the lengths of the sides be related in this way? Work with your team to look more closely at side lengths of some right triangles.

a. On centimeter graph paper, form a right angle by drawing one 5-cm length and one 12-cm length as shown at right. If you do not have centimeter graph paper, then use any graph paper to draw and measure these lengths with a ruler. After drawing the two lengths, create a right triangle by connecting the ends of the two lengths with a third side.

b. With a ruler, measure the longest side of the triangle in centimeters and label this length. If you do not have a centimeter ruler or you are using another kind of graph paper, create your triangle using 5 and 7 grid units. Then use an edge of the page and the grid lines as your ruler.

c. Visualize a square connected to each side of the right triangle in part (b). On your paper, sketch a picture like the one at right. What is the area of each square? Is the area of the square that is connected to the longest side equal to the sum of the areas of the other two squares?

d. Check this pattern with a new example.

- Draw a new right angle on the centimeter paper like you did in part (a). This time, use 9-cm and 12-cm lengths.
- Connect the endpoints to create a triangle, and measure the third side.
- Create a sketch for this triangle like the one you created in part (c), and find the areas of the squares.

Is the area of the square that is connected to the longest side equal to the sum of the other two areas?

e. The two shortest sides of a right triangle are called the legs, and the longest side is called the hypotenuse.

You previously wrote a statement about the relationship between the areas of squares drawn on the sides of a right triangle. Now use words to describe the relationship between the lengths of the legs and the length of the hypotenuse.
9-70. The relationship you described in part (e) of problem 9-69 is called the **Pythagorean Theorem**. It states that in a right triangle, the length of one leg squared plus the length of the other leg squared is equal to the length of the hypotenuse squared. It can be written as an equation like this:

\[(\text{leg A})^2 + (\text{leg B})^2 = (\text{hypotenuse})^2\]

Use the Pythagorean Theorem to write an equation for each diagram below. Then find each missing area.

a. \(9 \text{ cm}^2 \quad ? \text{ cm}^2 \quad 34 \text{ cm}^2\)

b. \(? \text{ m}^2 \quad 12 \text{ m}^2 \quad 8 \text{ m}^2\)

c. \(25 \text{ ft}^2 \quad 10 \text{ ft}^2 \quad ? \text{ ft}^2\)
9-71. In Lesson 9.2.1 you found a relationship between the squares of the sides of triangle and the type of triangle (acute, obtuse, or right). You discovered that if the sum of the squares of the two shortest sides in a triangle equals the square of the length of the longest side, then the triangle is a right triangle. Use this idea to determine whether the lengths listed below form a right triangle. Explain your reasoning.

a. 15 feet, 36 feet, and 39 feet  
b. 20 inches, 21 inches, and 29 inches  
c. 8 yards, 9 yards, and 12 yards  
d. 4 meters, 7 meters, and 8 meters
9-72. Find the area of the square in each picture below.

a. \[ \begin{array}{c}
11 \text{ m} \\
3 \text{ m}
\end{array} \]

b. \[ \begin{array}{c}
8 \text{ in.} \\
6 \text{ in.}
\end{array} \]

c. \[ \begin{array}{c}
5 \text{ ft} \\
4 \text{ ft}
\end{array} \]

d. \[ \begin{array}{c}
a \\
b
\end{array} \]
9-73. How long is the missing side of each triangle in parts (b) and (c) of problem 9-72? Be prepared to explain your reasoning.
The Triangle Inequality establishes the required relationships for three lengths to form a triangle. You can also use these lengths to determine the type of triangle they form — acute, obtuse, or right — by comparing the squares of the lengths of the sides as described below.

For any three lengths to form a triangle, the sum of the lengths of any two sides must be greater than the length of the third side.

For example, the lengths 3 cm, 10 cm, and 11 cm will form a triangle, because:

\[3 + 10 > 11\]
\[3 + 11 > 10\]
\[10 + 11 > 3\]

The lengths 5 m, 7 m, and 15 m will not form a triangle, because \(5 + 7 = 12\), and \(12 \neq 15\).

**Acute triangle:** The sum of the squares of the lengths of the two shorter sides is greater than the square of the length of the longest side.

**Obtuse triangle:** The sum of the squares of the lengths of the two shorter sides is less than the square of the length of the longest side.

**Right triangle:** The sum of the squares of the lengths of the two shorter sides is equal to the square of the length of the longest side.
9-74. If you have 24 square tiles, how many different rectangles can you make? Each rectangle must use all of the tiles and have no holes or gaps. Sketch each rectangle on graph paper and label its length and width. Can you make a square with 24 tiles?

9-75. Lydia has four straws of different lengths, and she is trying to form a right triangle. The lengths are 8, 9, 15, and 17 units. Which three lengths should she use? Justify your answer.

9-76. The Wild West Frontier Park now offers an unlimited day pass. For $29.00, visitors can go on as many rides as they want. The original plan charged visitors $8.75 to enter the park, plus $2.25 for each ride. Write an equation to determine the number of rides that would make the total cost equal for the two plans. Solve the equation.

9-77. a. Write the rule for the table at right.

<table>
<thead>
<tr>
<th>x</th>
<th>4</th>
<th>$\frac{1}{2}$</th>
<th>-2</th>
<th>-1</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-11</td>
<td>1</td>
<td>-3</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

b. What is the slope?

c. What is the y-intercept?

9-78. Solve for \( x \). Each part is a separate problem.

a. If \( m\angle 1 = 3x - 18^\circ \) and \( m\angle 5 = 2x + 12^\circ \), find \( x \).

b. If \( m\angle 3 = 4x - 27^\circ \) and \( m\angle 6 = x + 39^\circ \), find \( x \).

c. If \( m\angle 4 = 49^\circ \) and \( m\angle 6 = 5x + 41^\circ \), find \( x \).

9-79. Calculate the value of \( x \).

a.

b.
9.2.3 How can I find the side length?

Understanding Square Root

You have developed a way to decide if a triangle is a right triangle by looking at the squares of the side lengths of the triangle. If you already know a triangle is a right triangle, how can the Pythagorean Theorem help you determine the length of a leg or the hypotenuse? Today you and your team will develop new ways to find missing lengths of right triangles.

9-80. Nikita wants to use the area of the squares in the figure at right to find the lengths of the sides of a right triangle.

a. Find the missing area.

b. What are the lengths of the legs of the right triangle in Nikita’s diagram? How do you know?

c. About how long is the hypotenuse? Are you able to find the length exactly? Explain your reasoning.
Chapter 9-81

The numbers 36, 64, 4, 16, 100, 144, 121, and 225 are all examples of perfect squares.

a. If each of these numbers represents the number of square units in a square, what is the side length of each square?

b. Why do you think these numbers are called perfect squares?
9-82. To find the side length of a square with a particular area, you use an operation called the **square root**. The square root symbol looks like this: √. It is also called a **radical sign**.

To find the side length of a square with an area of 81 square units, for example, you would write √81 and would read it as “the square root of 81.” Since 9 · 9 = 81, then √81 = 9.

Copy each square root expression below. Rewrite each square root as an equivalent expression without the radical sign. Explain your method for finding the square root of these numbers.

a. √49  
   b. √121  
   c. √9  
   d. √169
9.83. In problem 9.74, you tried to make a perfect square with 24 tiles and could not.

a. Why was it not possible?

b. Estimate the length of a side of a square with an area of 24 square units. What two whole numbers is the length between?

c. Is \( \sqrt{24} \) closer to one of the whole numbers or to the other? If you did not already do so, estimate to the nearest tenth.

d. Multiply your estimate by itself. How close to 24 is your answer? If you revised your estimate, how would you change it?
9-84. Between which two whole numbers is each of the following square roots? To which whole number do you think it is closer? Estimate the value of the square root to the nearest tenth (0.1). You may find it helpful to create a list of the whole numbers from 1 to 17 and their squares to use with this kind of problem.

a. \( \sqrt{40} \)  
   b. \( \sqrt{95} \)  
   c. \( \sqrt{3} \)

d. \( \sqrt{59} \)  
   e. \( \sqrt{200} \)  
   f. \( \sqrt{154} \)

g. Describe your method for estimating the approximate value of a square root when the number is not a perfect square. Check each estimate for parts (a) through (f) on a calculator.
9-85. ESTIMATING WITH A GRAPH

In Chapters 6, 7, and 8, you examined the graphs of various relationships. What would the relationship between the side length of a square and the area of a square look like on a graph?

a. On the Lesson 9.2.3 Resource Page, complete the table for the side lengths and areas. Graph the points. Does it make sense to connect them? If so, connect them with a smooth curve.

b. Describe the relationship between the side length of a square and the area of the square. How is it the same or different than other relationships you have graphed?

c. How can you use the graph to estimate the side length for a square with an area of 24 square units? Does this estimate match your estimate in problem 9-83?

d. Use your graph to estimate these square roots:
   
i. $\sqrt{10}$   ii. $\sqrt{15}$   iii. $\sqrt{5}$   iv. $\sqrt{33}$
9-86. **Additional Challenge:** Nikita wonders, "*What can we say about the square root of a negative number?*" Discuss this question with your team. For example, can you find $\sqrt{-16}$? Write an explanation of your thinking. Be ready to share your ideas with the class.
LEARNING LOG

What is a square root? How can you estimate a square root? In your Learning Log, write directions for a fifth grader to follow. Explain what a square root is and how to estimate a square root to the nearest tenth. Include examples of perfect squares and non-perfect squares. Title this entry “Square Roots” and label it with today’s date.
METHODS AND MEANINGS

Right Triangles and the Pythagorean Theorem

A right triangle is a triangle in which the two shorter sides form a right (90°) angle. The shorter sides are called legs. The third and longest side, called the hypotenuse, is opposite the right angle.

The Pythagorean Theorem states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

\[(\text{leg 1})^2 + (\text{leg 2})^2 = (\text{hypotenuse})^2\]

Example:

\[3^2 + 4^2 = x^2\]
\[9 + 16 = x^2\]
\[25 = x^2\]
\[5 = x\]

The converse of the Pythagorean Theorem states that if the sum of the squares of the lengths of the two shorter sides of a triangle equals the square of the length of the longest side, then the triangle is a right triangle. For example:

Do the lengths 6, 9, and 11 form a right triangle?
\[6^2 + 9^2 \neq 11^2\]
\[36 + 81 \neq 121\]
No, these lengths do not form a right triangle.

Do the lengths 9, 40, and 41 form a right triangle?
\[9^2 + 40^2 = 41^2\]
\[81 + 1600 = 1681\]
Yes, these lengths form a right triangle.
9-88. Find the missing length or area.

\[
\begin{align*}
\text{a. } & \quad \frac{x}{225} \text{ cm} \quad 11 \text{ cm} \\
\text{b. } & \quad \square \\
\text{c. } & \quad \frac{100 \text{ m}^3}{w} \quad 5 \text{ m} \\
\text{d. } & \quad \frac{y}{150} \text{ ft}^2 \\
\end{align*}
\]

9-89. Determine the positive value that makes each equation true. If the answer is not a whole number, write it as a square root, and then approximate it as a decimal rounded to the nearest tenth.

\[
\begin{align*}
\text{a. } & \quad \text{If } x^2 = 36, x = ? \\
\text{b. } & \quad \text{If } x^2 = 65, x = ? \\
\text{c. } & \quad \text{If } x^2 = 84, x = ? \\
\text{d. } & \quad \text{If } x^2 = 13, x = ? \\
\end{align*}
\]

9-90. Use the rule \( y = -2x + 5 \) to answer the questions below.

\[
\begin{align*}
\text{a. } & \quad \text{What is the slope of the line?} \\
\text{b. } & \quad \text{Where does the line cross the y-axis?} \\
\end{align*}
\]

9-91. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.

\[
\begin{align*}
\text{a. } & \quad \frac{1.5}{2} \quad 3.5 \\
\text{b. } & \quad 0.2 \quad 17 \\
\text{c. } & \quad 2.5 \quad 0.2 \\
\text{d. } & \quad 10 \quad 4.5 \\
\end{align*}
\]

9-92. Identify which of the relationships shown below is not a function. Explain your reasoning.

\[
\begin{align*}
\text{a. } & \quad \begin{array}{ccccccc}
& 3 & 8 & 1 & 9 & -1 & 0 \\
\hline
x & 12 & 4 & 0 & 4 & -3 & -8 \\
\end{array} \\
\text{b. } & \quad \begin{array}{ccccccc}
& 5 & 2 & -1 & 0 & -15 & 2 \\
\hline
x & 2 & 0 & -11 & 8 & -25 & 1 \\
\end{array}
\end{align*}
\]

9-93. Kenneth claims that \((2, 0)\) is the point of intersection of the lines \( y = -2x + 4 \) and \( y = x - 2 \). Is he correct? How do you know?

9-94. Daniel needed to paint his patio, so he made a scale drawing of it. He knows that the width of the patio is 10 feet, but the scale drawing is in inches.

\[
\begin{align*}
\text{a. } & \quad \text{Find the length of the patio in feet.} \\
\text{b. } & \quad \text{Find the area of the patio so Daniel knows how much area he needs to paint.} \\
\text{c. } & \quad \text{One can of paint covers 125 square feet. How many cans of paint will Daniel need to buy?}
\end{align*}
\]
9-95. Juan found that 20 new pencils weigh 12 ounces. How much will 50 new pencils weigh? Show your reasoning.

9-96. For each diagram below, solve for $x$. Explain what method you used for each problem.

a. \[ \begin{align*}
   6x & \quad 4x + 10^\circ \\
   \end{align*} \]

b. \[ \begin{align*}
   5x + 13^\circ & \quad 3x + 7^\circ \\
   \end{align*} \]

c. \[ \begin{align*}
   3x + 5^\circ & \quad 2x + 18^\circ \quad 2x + 17^\circ \\
   \end{align*} \]

d. \[ \begin{align*}
   x & \quad 30^\circ \\
   \end{align*} \]

9-97. Find the surface area and volume of the rectangular prism at right.

9-98. Find the area of each circle below.

a. \[ \text{radius} = 8 \text{ cm} \]

b. \[ \text{radius} = 60 \text{ cm} \]

9-99. Write each number in scientific notation.

a. 49.63 \hspace{1cm} b. 0.0000005 \hspace{1cm} c. 3,120,000,000
Lesson 9.2.3 Resource Page

Graphing $y = x^2$

<table>
<thead>
<tr>
<th>Side Length $x$</th>
<th>Area of Square $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
9.2.4 What kind of number is it?

Real Numbers

Any number that can be written as the ratio of two integers \( \frac{a}{b} \) with \( b \neq 0 \) is called a rational number. A rational number can be matched to exactly one point on a number line. There are many other points on the number line, however, for which there is not a corresponding rational number. These numbers are called irrational numbers. Numbers such as \( \pi, \sqrt{2}, \) and \( -\sqrt{5} \) are irrational numbers. The rational numbers and the irrational numbers make up all of the numbers on the number line and together are called the real numbers.

In this lesson you will learn how to identify a number as rational or irrational. You will also write decimals as fractions to show that they are rational. Then you will compare these kinds of numbers and place them on the number line.

9-100. In previous courses, you worked with decimals that repeated and terminated. All of these are called rational numbers because they can be written as a ratio, like \( \frac{2}{3} \) and \( \frac{1}{2} = 0.5 \). Because \( \sqrt{9} = 3 \), \( \sqrt{9} \) is also a rational number.

However, there are some numbers that do not repeat or terminate when they are written as decimals, such as \( \sqrt{2} = 1.41421356237... \). Such numbers are called irrational numbers. An irrational number cannot be written as a ratio of any two integers. In other words, an irrational number cannot be written as a fraction.

Use your calculator to find the square root of the following numbers. Decide whether the decimals are rational (having decimals that terminate or repeat) or irrational.

a. \( \sqrt{6.25} \)  
b. \( \sqrt{100} \)  
c. \( \sqrt{7} \)
9-101. Do you think that you can decide by looking at it whether a number is rational or irrational? You will explore this idea in parts (a) through (d) below.

a. Without doing any calculations, which of the numbers below do you think are rational numbers? Which do you think are irrational numbers? Discuss this with your team and make predictions.

\[-\frac{5}{9} \quad \sqrt{7} \quad \frac{21}{4} \quad -\sqrt{15}\]

\[\pi^2 \quad -\sqrt{76} \quad \frac{730}{99} \quad -6.4 \times 10^{-2}\]

b. Now use your calculator and write the equivalent decimal for each of the numbers in the list. Were your predictions correct?

c. What do you notice about the decimal forms of rational numbers compared to irrational numbers?

d. Is \(\sqrt{42.25}\) rational or irrational? Explain your answer.
9-102. Every rational number can be written as a fraction, that is, as a ratio of two integers. Since 0.78 is described in words as "seventy-eight hundredths," it is not a surprise that the equivalent fraction is \( \frac{78}{100} \). Use what you know about place value to rewrite each terminating decimal as a fraction. Check your answers with a calculator.

a. 0.19  
b. 0.391  
c. 0.001  
d. 0.019  
e. 0.3  
f. 0.524
9-103. Jessica knows that 0.57 is a rational number, so she should be able to write it as a fraction. She wonders how to rewrite it, though. She started to rewrite it as $\frac{57}{100}$, but she is not sure if that correct. Is $\frac{57}{100}$ equal to 0.57? Be ready to justify your answer.
9-104. To help Jessica with her problem, find the decimal equivalents for the fractions below.

a. \( \frac{19}{99} \)  

b. \( \frac{391}{999} \)  

c. \( \frac{3}{9} \)  

d. \( \frac{1}{999} \)  

e. \( \frac{524}{999} \)  

f. \( \frac{16}{999} \)  

g. What patterns do you see between the fractions and their equivalent decimals? What connections do these fractions have with those you found in problem 9-102? Be ready to share your observations with the class.

h. Use your pattern to predict the fraction equivalent for 0.24. Then test your guess with a calculator.

i. Use your pattern to predict the decimal equivalent for \( \frac{65}{99} \). Check your answer with your calculator.
REWRITING REPEATING DECIMALS AS FRACTIONS

Jessica wants to figure out why the pattern from problem 9-104 works. She noticed that she could eliminate the repeating digits by subtracting, as she wrote at right. This gave her an idea. "What if I multiply by something before I subtract, so that I'm left with more than zero?" she wondered. She wrote:

\[
10(0.\overline{57}) = 5.75757... \\
- (0.\overline{57}) = -0.575757...
\]

"The repeating decimals don't make zero in this problem. But if I multiply by 100 instead, I think it will work!" She tried again:

\[
100(0.\overline{57}) = 57.575757... \\
- (0.\overline{57}) = -0.575757... \\
99(0.\overline{57}) = 57.0
\]

a. Discuss Jessica's work with your team. Why did she multiply by 100? How did she get 99 sets of 0.\overline{57}? What happened to the repeating decimals when she subtracted?

b. "I know that 99 sets of 0.\overline{57} are equal to 57 from my equation," Jessica said. "So to find what just one set of 0.\overline{57} is equal to, I will need to divide 57 into 99 equal parts." Represent Jessica's idea as a fraction.

c. Use Jessica's strategy to rewrite 0.\overline{98} as a fraction. Be prepared to explain your reasoning.
9-106. Show that the following repeating decimals are rational numbers by rewriting them as fractions.

a. $0.\overline{42}$    b. $0.\overline{312}$    c. $0.\overline{16}$    d. $0.\overline{8}$
9-107. Indicate the approximate location of each of the following real numbers on a number line. What can make this task easier? Try to do it without using a calculator.

\[ \frac{2}{3}, -0.75, \sqrt{8}, -\frac{9}{5}, \frac{\pi}{3}, 2 \frac{1}{4} \]
9-108. Without using a calculator, order the numbers below from least to greatest.

\[ \sqrt{102}, 10, 3\pi, \sqrt{99}, 1.1 \times 10^1, 9.099 \]
9-109. Copy and complete the following sentences.

a. The set of all numbers on the number line are called the ____________.

b. A number that has an equivalent terminating or repeating decimal is called a(n) ____________.

c. A number that has an equivalent decimal that is non-repeating is called a(n) ____________.

d. Any number that can be written as a fraction of integers is a(n) ____________.
METHODS AND MEANINGS

The Real-Number System

The **real numbers** include all of the **rational numbers** and **irrational numbers**.

**Rational numbers** are numbers that can be written as a fraction in the form \( \frac{p}{q} \), where \( p \) and \( q \) are integers and \( q \neq 0 \). Rational numbers written in decimal form either terminate or repeat. The number 7 is a rational number, because it can be written as \( \frac{7}{1} \). The number \(-0.687\) is rational, because it can be written as \( -\frac{687}{1000} \). Even \( \sqrt{25} \) is rational, because it can be written as \( \frac{5}{1} \). Other examples of rational numbers include \(-12, 0, 3, \frac{1}{8}, \frac{5}{9}, 0.25, \) and \( \sqrt{81} \).

**Irrational numbers** are numbers that cannot be written as fractions. Decimals that do not terminate or repeat are irrational numbers. For example, \( \sqrt{3} \) is an irrational number. It cannot be written as a fraction, and when it is written as a decimal, it neither terminates nor repeats (\( \sqrt{3} \approx 1.73205080756\ldots \)). Other irrational numbers include \( \sqrt{2}, \sqrt{7}, \) and \( \pi \).
9-110. Graph each of the pairs of points listed below and draw a line segment between them. Use the graph to help you find the length of each line segment. State whether each length is irrational or rational.

a. \((-3,0)\) and \((0,-3)\)  
b. \((2,3)\) and \((-1,2)\)  
c. \((3,2)\) and \((3,-3)\)

9-111. Howie and Steve are making cookies for themselves and some friends. The recipe they are using will make 48 cookies, but they only want to make 16 cookies. They have no trouble reducing the amounts of flour and sugar, but the original recipe calls for \(1\frac{3}{4}\) cups of butter. Help Howie and Steve determine how much butter they need.

9-112. Find the perimeter and area of the figure at right.
Copy the figure on your paper, and show your work for each of the steps that you use.

9-113. Write the following numbers in scientific notation.

a. 370,000,000  
b. 0.000000000076

9-114. Simplify each of the following expressions.

a. \(4x^3y \cdot 3xy^2\)  
b. \(6a^5b^2 \cdot 3ab^2\)  
c. \(m^2n \cdot 9mn\)

d. \(\frac{3^5 \cdot 8 \cdot 3^3}{3^2 \cdot 2^3 \cdot 3^5 \cdot 3^3}\)  
e. \(\frac{m^4n}{n^3}\)  
f. \(\frac{2a^4b^2}{15b}\)

9-115. Simplify each numerical expression.

a. \(|5 - 6 + 1|\)  
b. \(-2\cdot|16|\)  
c. \(|6 - 2| + |8 - 1|\)
9-116. Identify the following numbers as rational or irrational. If the number is rational, show that it can be written as a fraction.

a. $\sqrt{36}$
b. $0.\overline{62}$
c. $\sqrt{92}$

9-117. Solve each system.

a. $y = 2x + 1$
   $y = -3x - 4$
b. $y = \frac{1}{3}x + 4$
   $y = \frac{1}{2}x - 2$

9-118. For the rule $y = 6 + (-3)x$:

a. What is the y-intercept?  
b. What is the slope?

9-119. Make a table and graph the rule $y = \sqrt{x - 2}$ that includes $x$-values from $-1$ to $10$. Graph the rule on graph paper.

9-120. Dawn drove 420 miles in 6 hours on a rural interstate highway. If she maintains the same speed, how far can she go in 7.5 hours?

9-121. The attendance at the county fair was lowest on Thursday, the opening day. On Friday, 5500 more people attended than attended Thursday. Saturday doubled Thursday's attendance, and Sunday had 3000 more people than Saturday. The total attendance was 36,700. Write and solve an equation to find how many people attended the fair each day.
9.2.5 How can I find missing parts?

Applications of the Pythagorean Theorem

In this section, you have studied different properties of triangles. You have used the Pythagorean Theorem to describe the special relationship between the sides of a right triangle. In this lesson, you will use these ideas to solve a variety of different problems.

9-122. Ann is measuring some fabric pieces for a quilt. Use the Pythagorean Theorem and your calculator to help her find each of the missing lengths below. Decide whether each answer is rational or irrational. If it is rational, explain whether the decimal repeats or terminates.

a. 
\[
\begin{array}{c}
32 \text{ in.} \\
\text{24 in.}
\end{array}
\]

b. 
\[
\begin{array}{c}
10 \text{ m} \\
20 \text{ m}
\end{array}
\]

c. 
\[
\begin{array}{c}
7.6 \text{ cm} \\
9.5 \text{ cm}
\end{array}
\]
9-123. Coach Kelly’s third-period P.E. class is playing baseball. The distance between each base on the baseball diamond is 90 feet. Lisa, at third base, throws the ball to Dano, at first base. How far did she throw the ball? State whether your answer is rational or irrational.
9-124. As the city planner of Right City, you are responsible to report information to help the Board of Supervisors make decisions about the budgets for the fire and police departments. The board has asked for a report with answers to the following questions. Each grid unit in the figure at right represents 1 mile.

a. For fire safety, bushes will be cleared along the perimeter of the city. What is the length of the perimeter? Include all of your calculations in your report.

b. Some people say that Right City got its name from its shape. Is the shape of the city a right triangle? Show how you can tell.
9-125. Clem and Clyde have a farm with three different crops: a square field of corn, a rectangular field of artichokes, and a right-triangle grove of walnut trees (as shown at right). A fence totally surrounds the farm. Find the total area of Clem and Clyde’s land in square miles and tell them how much fencing they need to enclose the outside of their farm.
9-126. Scott and Mark are rock climbing. Scott is at the top of a 75-foot cliff, when he throws a 96-foot rope down to Mark, who is on the ground below. If the rope is stretched tightly from Mark's feet to Scott's feet, how far from the base of the cliff (directly below Scott) is Mark standing? Draw a diagram and label it. Then find the missing length. Is the length irrational?
9-127. Nicole has three long logs. She wants to place them in a triangle around a campfire to allow people to sit around the fire. The logs have lengths 19, 11, and 21 feet.

a. Can she form a triangle with these lengths? If so, what type of triangle (acute, obtuse, or right) will the logs form? Justify your answers.

b. Nicole realized she wrote the numbers down incorrectly. Her logs are actually 9, 11, and 21 feet. Will she still be able to surround her campfire with a triangular seating area? If so, will the shape be a right triangle? Justify.
9-128. LEARNING LOG

The Pythagorean Theorem describes the special relationship that exists between the side lengths of a right triangle. In your Learning Log, describe how the Pythagorean Theorem can be used to find a missing side length on a right triangle. Make up examples in which either the hypotenuse length is missing or a leg length is missing. Be sure to include pictures and describe how you would find each missing length. Title this entry "Pythagorean Theorem" and label it with today's date.
METHODS AND MEANINGS

Squaring and Square Root

When a number or variable is multiplied by itself, it is said to be squared. Squaring a number is like finding the area of the square with that number or variable as its side length. For example:

\[ 6 \cdot 6 = 6^2 = 36 \quad \text{and} \quad 36 \text{cm}^2 \quad 6 \text{ cm} \quad a \cdot a = a^2 \quad \text{and} \quad a^2 \quad a \]

6 cm

The square root of a number or variable is the positive factor that, when multiplied by itself, results in the given number. Use the radical sign, \( \sqrt{\text{\text{ }}\text{\text{}}} \), to show this operation. If you know the area of a square, then the square root of the numerical value of the area is the side length of that square.

For example, \( \sqrt{49} \) is read as, “the square root of 49,” and means, “Find the positive number that multiplied by itself equals 49.” \( \sqrt{49} = 7 \), since \( 7 \cdot 7 = 49 \).

By definition, \( -7 \) is not the square root of 49 even though \( (-7) \cdot (-7) = 49 \), since only consider positive numbers are considered to be square roots. No real square region could have a negative side length.
9-129. On a coordinate grid, draw a triangle with vertices at (2, 6), (2, 2), and (5, 6).
   a. Find the lengths of each side of the triangle. What is the perimeter?
   b. What type of triangle is formed by these points? Justify your answer.

9-130. Ann lives on the shoreline of a large lake. A market is located 24 km south and
32 km west of her home on the other side of the lake. If she takes a boat across
the lake directly toward the market, how far is her home from the market?

9-131. Change each number below from a decimal to an equivalent fraction. For help
with the repeating decimals, review the Math Notes box from Lesson 9.2.4.
   a. 0.7  b. 0.7  c. 0.15  d. 0.15

9-132. Simplify each of the following expressions.
   a. $3 \frac{1}{5} \cdot \frac{7}{4}$
   b. $5 \cdot (-\frac{4}{5})$
   c. $2^4 \times \frac{5}{8}$
   d. $-\frac{1}{2} \cdot 3^2$
   e. $-\frac{5}{6} + (\frac{1}{2})^2$
   f. $(-\frac{4}{5})^2 - \frac{3}{10}$
   g. $(\frac{4}{10})^2 - (-\frac{2}{5})^2$
   h. $8^2 \times (-\frac{7}{8}) - \frac{1}{2}$

9-133. Simplify each expression.
   a. $(3x)^4 \cdot x^3$
   b. $\frac{2^4 \cdot 3}{2^3 \cdot 3^2}$
   c. $4^{-3} \cdot 4^2$

9-134. Identify the type of growth in each table below as linear or non-linear.
   a. 
   
   \[
   \begin{array}{ccccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & -8 & -1 & 0 & 8 & 27 \\
   \end{array}
   \]
   
   b. 
   
   \[
   \begin{array}{ccccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & 4 & 6 & 8 & 10 & 12 & 14 \\
   \end{array}
   \]
   
   c. 
   
   \[
   \begin{array}{ccccccc}
   x & -2 & -1 & 0 & 1 & 2 & 3 \\
   y & \frac{1}{4} & \frac{1}{2} & 1 & 2 & 4 & 8 \\
   \end{array}
   \]
9.2.6 How can I find lengths in three dimensions?

Pythagorean Theorem in Three Dimensions

You have been studying right triangles from a two-dimensional viewpoint. However, often triangles occur in three-dimensional settings. In this lesson, you will build a model, use it to visualize the location of right triangles in a three-dimensional situation, and then make calculations to find the triangle lengths.

9.135. The roof on the Flat Family’s house is one large flat rectangle. It is parallel to the ground. The TV antenna is mounted in the center of the roof. Guy wires are attached to the antenna five feet below its highest point, and they are attached to the roof at each corner and at the midpoint of each edge. These wires support the antenna in the wind.

Each guy wire is the hypotenuse of a right triangle. Mr. Flat needs to know how long each wire needs to be to keep the antenna upright. He also needs to find the total amount of wire needed.

a. Build a model of the Flat Family Roof. The model will serve as a reference while your team finds the lengths of the sides of the right triangles in a three-dimensional setting.

1. For the roof, use a rectangular piece of cardboard. Draw the diagonals of the rectangle to locate the center.

2. Tape the string guy wires to the antenna (straw). Do not forget to leave a gap that represents the five feet between the strings and the top of the antenna. How many guy wires are there?

3. Attach the antenna to the center of the roof. Use tape, tie knots, or make slits to anchor your antenna and guy wires to the roof.

b. Locate as many vertical right triangles as you can on your model. How many right triangles did you find?

c. Identify which right triangles are the same size and which are different sizes. How many different-sized right triangles did you find? How many of each size are there?

d. Sketch each of the different-sized right triangles on a different color of paper. Cut out the triangle. Tape the triangle in position on your model.
9-136. The Flat Family's roof is 60 feet long and 32 feet wide. The TV antenna is 30 feet tall. The wires are attached five feet below the top of the antenna.

a. How long must each guy wire be? Show all of your steps to get your solution.

b. Mr. Flat needs to buy the wire at his local hardware store. He decides to buy an additional 10% more than the amount that you calculated so that he can attach the wires on each end. How much total wire should he buy? Show all of your work so that Mr. Flat can understand it, and round your answer reasonably.
9-137. A child’s shoe box measures 4" by 6" by 3". What is the longest pencil you could fit into this box? An empty box may help you visualize the various ways you could fit the pencil in the box. If possible, draw a diagram to show the pencil’s position. Show your steps.
9-138. **Additional Challenge:** A fly was sitting on the ground in the back left corner of your classroom. He flew to the ceiling in the opposite corner of the room. How far did he fly if he went in a straight line? Draw a diagram of your classroom, complete with the correct measurements. Write equations. Show your steps.
9-139. The hypotenuse and one leg of a right triangle are 65 and 60 meters. What is the length of the third side?

9-140. Find the missing length on this right triangle.

9-141. Solve these equations for $x$.
   a. $(x + 3.5)2 = 16$  
   b. $23 + 5x = 7 + 2.5x$  
   c. $3x + 4.4 = -(6.6 + x)$

9-142. Simplify each expression.
   a. $\frac{3}{2} \cdot \frac{5}{3}$  
   b. $10x^4(10x)^{-2}$  
   c. $(\frac{1}{4})^3 \cdot (4)^2$  
   d. $\frac{(xy)^3}{xy^3}$

9-143. Complete the table.
   
<table>
<thead>
<tr>
<th>$x$</th>
<th>-5</th>
<th>1</th>
<th>3</th>
<th>0</th>
<th>8</th>
<th>-2</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>7</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

   a. Find the rule.
   b. What is the slope?

9-144. Additional Challenge: Use the diagram and your answer to problem 9-138 above. If your classroom measured 30 ft by 30 ft by 10 ft, what distance would the fly travel?
9.2.7 Does it always work?

Pythagorean Theorem Proofs

You have seen that you can find the missing side of a right triangle using the Pythagorean Theorem. To show that it is always true, no matter how long the sides are, it must be proven. There are over 100 different ways to prove this important relationship. Today you will look at two ways to prove the Pythagorean Theorem. As you may remember, the Pythagorean Theorem states that in a right triangle with sides $a$ and $b$, and hypotenuse $c$, $a^2 + b^2 = c^2$.

9-145. PYTHAGOREAN THEOREM PROOF

Obtain the Lesson 9.2.7A Resource Page from your teacher. Start by cutting out four copies of the right triangle with legs labeled $a$, $b$, and hypotenuse labeled $c$ units.

a. First, arrange the triangles to look like the diagram at right. Draw this diagram on your paper. Explain why the area of the unshaded region is $c^2$.

b. Will moving the triangles within the bold outer square change the total unshaded square?

c. Move the shaded triangles to match the diagram at right. In this arrangement, tell why the total area that is unshaded is $a^2 + b^2$.

d. Write an equation that relates the unshaded region in part (a) to the unshaded region in part (b).
9-146. Here is another proof of the Pythagorean Theorem for you to try. Obtain the Lesson 9.2.7B Resource Page from your teacher.

a. Start with two squares as shown at right. What is the total area?

b. Use a ruler or another piece of paper to place a mark of length $b$ on the bottom left side of the larger square. Then draw the dotted lines as shown in the diagram at right to create two right triangles. How do you know that the legs of both triangles are legs $a$ and $b$? Label each hypotenuse $c$.

c. Cut out the shaded triangles shown in the diagram at right. Then work with your team to determine how to arrange the shaded triangles and the unshaded portion of the original figure to create a square. What is the area of the square? How do you know?

d. How does what you have done in this problem prove the Pythagorean Theorem?
9-147. The **converse** of a theorem reverses the evidence and the conclusion. The Pythagorean Theorem states that in a right triangle with legs of \( a \) and \( b \), and hypotenuse \( c \), that \( a^2 + b^2 = c^2 \).

a. State the converse of the Pythagorean Theorem.

b. Look back at your work from problem 9-51. What can you conclude about a triangle if \( a^2 + b^2 = c^2 \)?

c. Why is this not a *proof* of the converse of the Pythagorean Theorem?
9-148. Graph the points \( A (-2, 2) \) and \( B (1, -2) \). Then find the distance between them by creating a right triangle (like a slope triangle) and computing the length of the hypotenuse.
9-149. Use a graph to find the distance between the points \( C (-4, -1) \) and \( D (4, 1) \).

9-150. Jack has a tree in his backyard that he wants to cut down to ground level. He needs to know how tall the tree is, because when he cuts it, it will fall toward his fence. Jack measured the tree's shadow, and it measured 20 feet long. At the same time, Jack's shadow was 12 feet long. Jack is 5 feet tall.

a. How tall is the tree?

b. Will the tree hit the fence if the fence is 9 feet away?

9-151. Examine the diagrams below. What is the geometric relationship between the labeled angles? What is the relationship of their measures? After you determine the relationship of their measures, use the relationship to write an equation and solve for \( x \).

\[
\begin{align*}
a. & \quad 3x + 5^\circ \\
& \quad 5x - 57^\circ \\
b. & \quad 4x + 150^\circ \\
& \quad 2x^\circ \\
\end{align*}
\]

9-152. On graph paper, graph the system of equations at right.

\[
\begin{align*}
y &= -\frac{2}{5}x + 1 \\
y &= -\frac{3}{2}x - 2
\end{align*}
\]

Then state the solution to the system. If there is not a solution, explain why not.

9-153. A principal made the histogram at right to analyze how many years teachers had been teaching at her school.

\[
\text{Frequency}
\]

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Years of Teaching} & 0 & 5 & 10 & 15 & 20 & 25 \\
\hline
\text{Frequency} & 1 & 3 & 4 & 5 & 2 & 1 \\
\hline
\end{array}
\]

a. How many teachers work at her school?

b. If the principal randomly chose one teacher to represent the school at a conference, what is the probability that the teacher would have been teaching at the school for more than 10 years? Write the probability in two different ways.

c. What is the probability that a teacher on the staff has been there for fewer than 5 years?

9-154. Simplify each expression.

\[
\begin{align*}
a. & \quad \frac{12}{5} + \frac{7}{10} \\
b. & \quad \frac{9}{4} + (-\frac{1}{3}) \\
c. & \quad -\frac{1}{2} + (-\frac{1}{6})
\end{align*}
\]
Lesson 9.2.7A Resource Page

Diagram of geometric figures with labeled sides.
Chapter 9 Closure  What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel comfortable with, ideas that you would like to learn more about, and topics with which you need more help.

SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about properties of triangles and applying the Pythagorean Theorem, two of the main ideas of this chapter.

Triangular Treasure Hunt

Jasmine and Mason are spending the summer with their Uncle Simon. He lives in an old castle that has many interesting doors, not all of which are rectangles. One day, while looking through the books in the castle library, Jasmine found an unusual piece of paper in a book. “Look, Mason! This looks like some kind of a treasure map,” she exclaimed. Mason looked at the paper and read, “A valuable secret is hidden inside. But beware – danger lurks behind the other doors. Only those who can follow the clues will succeed in the search.” He also saw that it had a series of clues and pictures of triangular doors. Read the clues below and, with your team, help Jasmine and Mason find the secret door and the treasure.

- My sides are not congruent.
- My angles are not congruent.
- I am similar to another door.
- One of my sides is $\sqrt{41}$ units long.

a. Look at the doors on the Chapter 9 Closure Resource Page.

b. Using what you have learned in this chapter, decide which door is the secret door. Justify your answer.

c. List the triangles that were not the secret door and justify why each did not fit the clues.
WHAT HAVE I LEARNED?

Doing the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

Solve each problem as completely as you can. The table at the end of this closure section provides answers to these problems. It also tells you where you can find additional help and where to find practice problems like them.
CL 9-155. Solve for $x$.

a. 

$$128^\circ$$

$$10x + 2^\circ$$

b. 

$$80^\circ$$

$$65^\circ$$

CL 9-156. Solve for $x$.

a. 

$$5x + 3^\circ$$

$$52^\circ$$

$$128^\circ$$

b. 

$$x^\circ$$

$$58^\circ$$

$$23^\circ$$
CL 9-157. Determine the length of side \( x \). Give the answer in radical form and as a decimal.

CL 9-158. Simplify the following exponential expressions. Give answers without negative exponents.

a. \( 4^{-3} \cdot 4^7 \)

b. \( (5x^4)^3 \)

c. \( \frac{3^7}{3^4} \)

d. \( (4x^5)(3x^{-8}) \)
CL 9-159. Visualize a line that goes through the two points on the graph at right.

a. What is the slope of the line?

b. What is the rule for the line?

CL 9-160. Casey was building a rectangular pen for his pigs. He has 62 feet of fencing. The length of his pen is 9 feet longer than the width. Write and solve an equation to find the dimensions of the pen.
CL 9-161. Clay and his friend Lacey are making cookies for the school dance. Clay started early and has already made 3 dozen cookies. He can make an additional 2 dozen cookies an hour. Lacey has not started making cookies yet, but she has a bigger oven and can make 4 dozen cookies an hour. If Lacey starts baking her cookies right away, how long will it take for her to have made as many cookies as Clay? How many cookies will they each have made?

CL 9-162. Sarah loves to order paperback mystery books. Some of her recent orders are shown in the table below.

<table>
<thead>
<tr>
<th># of Books</th>
<th>1</th>
<th>3</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost</td>
<td>10.25</td>
<td>18.00</td>
<td>10.25</td>
<td>21.75</td>
<td>14.00</td>
<td>26.25</td>
</tr>
</tbody>
</table>

a. Draw a scatterplot with this information.

b. Fully describe the association.

c. Determine the equation of the line of best fit.

d. Based on the equation, estimate the total cost of an order of 10 books.
CL 9-163. For each of the problems above, do the following:

- Draw a bar or number line that represents 0 to 10.

![Number Line Image]

- Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose one of those problems and do one of the following tasks:

- Write two questions that you would like to ask about that problem.
- Brainstorm two things that you DO know about that type of problem.

If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:

- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
- Make a new problem that is similar and more challenging than that problem and solve it.
WHAT TOOLS CAN I USE?

You have created or have available to you several tools and references that help support your learning – your teacher, your study team, your math book and your Toolkit to name a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update your Toolkit to better reflect your current understanding of big ideas.

Listed below are the main elements of your Toolkit, Learning Log Entries, Methods and Meanings Boxes, and vocabulary used in this chapter. The words that appear in bold are new to this chapter. Use these lists and follow your teacher’s instructions to ensure that your Toolkit is a useful tool as well as a complete reference for you as you complete this chapter and prepare to begin the next one.

Learning Log Entries
- Lesson 9.1.1 – Angle Relationships
- Lesson 9.1.2 – Angles in a Triangle
- Lesson 9.1.4 – Angle-Angle Similarity
- Lesson 9.2.1 – Triangle Inequality
- Lesson 9.2.1 – Triangle Side-Length Patterns
- Lesson 9.2.3 – Square Roots
- Lesson 9.2.5 – Pythagorean Theorem

Math Notes
- Lesson 9.1.1 – Angle Vocabulary
- Lesson 9.1.2 – Parallel Lines and Angle Pairs
- Lesson 9.1.3 – Angle Sum Theorem for Triangles
- Lesson 9.1.4 – Exterior Angle Theorem for Triangles
- Lesson 9.2.1 – AA Similarity for Triangles
- Lesson 9.2.2 – Triangle Inequality and Side-Length Patterns
- Lesson 9.2.3 – Right Triangles and the Pythagorean Theorem
- Lesson 9.2.4 – The Real Number System
- Lesson 9.2.5 – Squaring and Square Root

Mathematical Vocabulary
The following is a list of vocabulary found in this chapter. Some of the words have been seen in previous chapters. The words in bold are the words new to this chapter. Make sure that you are familiar with the terms below and know what they mean. For the words you do not know, refer to the glossary or index. You might also add these words to your Toolkit so that you can reference them in the future.

AA~ (Angle-Angle Similarity)  acute angle
adjacent angles  alternate interior angles
complementary angles  corresponding angles
exterior angle  hypotenuse
leg (of a right triangle)  obtuse angle
parallel  perfect square
perpendicular  Pythagorean Theorem
radical sign remote interior angle
right angle  square (of a number)
square root  supplementary angles
straight angle  transversal
vertex  vertical angles
# Answers and Support for Closure Problems

*What Have I Learned?*

Note: MN = Math Note, LL = Learning Log

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>Need Help?</th>
<th>More Practice</th>
</tr>
</thead>
</table>
| CL 9-155. | a. 5  
  b. 35 | Lessons 9.1.2  
  and 9.1.3  
  MN: 9.1.2,  
  9.1.3, and  
  9.1.4  
  LL: 9.1.1,  
  9.1.2, and  
  9.1.4 | Problems 9-14,  
  9-18, 9-22, 9-28,  
  9-29, 9-30, 9-33,  
  9-34, 9-49, 9-78,  
  9-79, 9-82, 9-151,  
  and 9-154 |
| CL 9-156. | a. 25  
  b. 81 | Lessons 9.1.2  
  and 9.1.3  
  MN: 9.1.2,  
  9.1.3, and  
  9.1.4  
  LL: 9.1.1,  
  9.1.2, and  
  9.1.4 | Problem  
  CL 9-155 |
| CL 9-157. | \( \sqrt{16} = 13.27 \) | Lessons 9.2.2,  
  9.2.3, and  
  9.2.5  
  MN: 9.2.3 and  
  9.2.5  
  LL: 9.2.3 and  
  9.2.5 | Problems  
  9-70,  
  9-122, 9-140, and  
  9-139 |
| CL 9-158. | a. \( 4^4 \)  
  b. \( 125 \sqrt[12]{2} \)  
  c. \( 3^3 \)  
  d. \( \frac{12}{2^3} \) | Lesson 8.2.1  
  MN: 8.1.2 and  
  8.2.4  
  LL: 8.2.3 | Problem  
  CL 9-156 |
| CL 9-159. | a. The slope is \( \frac{1}{2} \).  
  b. \( y = \frac{1}{2}x \) | Lesson 7.2.4  
  LL: 7.3.1 | Problems  
  CL 7-122 and  
  9-90 |
| CL 9-160. | 2x + 2(x + 9) = 62  
  OR 4x + 18 = 62  
  The dimensions of the pen are  
  11 feet by 20 feet. | MN: 1.1.3 | Problem 9-59 |
| CL 9-161. | In 1 \( \frac{1}{2} \) hours, they will each have  
  made 72 cookies. | Lessons 5.2.2  
  and 5.2.3  
  MN: 5.2.2,  
  5.2.3, and  
  5.2.4 | Problems  
  9-48,  
  9-76, and 9-152 |
| CL 9-162. | a. See graph at  
  right.  
  b. Strong positive  
  association  
  c. \( y = 4x + 6 \) | Lesson 7.1.3  
  and 7.3.2 | Problems 9-10,  
  9-24, and 9-150 |
  d. \$46.00 | MN: 7.1.3 and  
  7.3.2  
  LL: 7.1.3 and  
  7.3.2 |
Chapter 9 Closure Resource Page

Triangular Treasure Hunt

A valuable secret is hidden inside.
Just knowing – danger lurks behind the other doors.
Only those who can follow the clues will unravel the mystery.

Clues:
- My sides are not congruent.
- My angles are not congruent.
- I am similar to another door.
- One of my sides is \( \sqrt{11} \) units long.

Diagram:

- Triangle A:
  - Sides: 4, \( \sqrt{41} \), 8.5

- Triangle B:
  - Sides: 8, 8, 14

- Triangle C:
  - Sides: 10, 8

- Triangle D:
  - Sides: \( \sqrt{41} \), \( \sqrt{41} \), \( \sqrt{41} \)

- Triangle E:
  - Sides: 4, 6

- Triangle F:
  - Sides: 5, 6, 5

- Triangle G:
  - Sides: 5, 4